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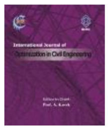

OPTIMIZATION-BASED SIMPLIFICATION OF A HIGH-RISE BENCHMARK STEEL BUILDING FOR DYNAMIC ANALYSIS

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ABSTRACT

Reducing the degrees of freedom of building models significantly reduces computational costs in time-consuming structural engineering problems, such as dynamic analysis, nonlinear analysis, or the optimal design of structural systems. In this study, the Finite Element (FE) model of a 20-story benchmark steel building with numerous degrees of freedom (DoF) is simplified to a 20-degree-of-freedom linear shear-type building. First, a preliminary linear shear-type model was derived by estimating the story stiffness so that the fundamental frequency matches that of the FE model. Then, an optimization problem is formulated and solved using a Genetic Algorithm (GA) combined with a weighted-sum method to achieve greater accuracy at higher frequencies in the preliminary model. Two objective functions were established and assessed for the optimization problem: one is the difference in frequencies between the FE model and the preliminary model with equal weighting, and the other is the first objective function improved with the modal participation percent weighting. The stiffness of each story in the preliminary model is selected as the design variable in both optimization problems. Finally, these optimized models are evaluated against the FE model using frequencies and dynamic time-history responses. The model derived from the weighted objective function demonstrates acceptable accuracy compared to its FE model in frequency and time-history analysis. It can be used for dynamic analysis and other structural and earthquake engineering purposes.

Keywords: Dynamic Analysis; Finite Element Model; Degrees of Freedom; Shear-Type Model; Genetic Algorithm; weighted-sum method; Benchmark Steel Building.

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1. INTRODUCTION

Engineering structures are being increasingly constructed on a larger scale, resulting in heightened structural complexity and more stringent design specifications. Various methodologies have been employed for analysis and design to address these evolving demands. This analysis involves the computation of displacements, deformations, and internal stresses within the structure over a specified time frame under defined dynamic loading conditions. Accordingly, the availability of reliable dynamic analysis tools is critical throughout the structural design process [1]. Dynamic analysis may be performed analytically for simple structural systems subjected to well-defined loading conditions. However, advanced computational approaches have become indispensable for more intricate structures and complex loading scenarios. The introduction of the Finite Element (FE) method [2] marked a paradigm shift in computational dynamic analysis. FE-based techniques are fundamental tools for structural analysis, design, and control applications.

Nevertheless, increasing accuracy in such methods comes at the expense of significant modeling complexity and elevated computational demands [3]. FE models often comprise hundreds of thousands of degrees of freedom (DoF), making the computation of dynamic responses extremely time-consuming and computationally expensive. When multiple design configurations and loading conditions must be considered, conducting comprehensive dynamic analyses within acceptable time constraints often becomes infeasible [4]. Thus, structural engineers must strike a balance between two competing objectives: achieving modeling accuracy and precision while mitigating time, cost, and computational burden.

Model Order Reduction (MOR) techniques offer an effective and practical strategy for approximating the original model with a substantially lower order. In essence, these methods produce a Reduced-Order Model (ROM) that can replicate the behavior of large-scale dynamic systems. This reduction leads to notable decreases in computation time, cost, and effort while preserving the accuracy of analytical results [5]. Given the widespread use of FE analysis in evaluating and designing structural systems, integrating MOR techniques has become increasingly essential in this domain. Reducing DoF in engineering structures serves a range of objectives, including seismic performance assessment, seismic demand estimation, structural health monitoring, damage detection and evaluation, and control system development. Structures such as buildings typically possess many DoF and are commonly simplified into reduced-order representations, such as single-degree-of-freedom (SDoF) or shear-type models.

Reducing DoF through mass and stiffness matrices condensation was introduced in 1965 [6]. Since then, numerous studies have investigated the use of ROMs as alternatives to full-scale FE models. One study implemented ROM updating iteratively to reduce the mass and stiffness matrices of a 40-story shear-type building, maintaining an acceptable level of accuracy consistently [7]. Another work presented a simplified approach for the preliminary design of structural dampers by approximating 3-, 9-, and 20-story benchmark steel buildings into shear-type models [8]. An iterative reduction method was also proposed to reduce 6- and 12-story concrete buildings into SDoF models [9]. A separate study simplified a 9-story benchmark steel building to an equivalent linear shear-type model by minimizing discrepancies in natural frequencies and mode shapes between the FE model and the ROM [10].

Further research introduced a method to reduce the DoF of high-rise shear-type structures, specifically 40- and 60-story models, by condensing them into 10- and 15-story equivalents [11]. Another approach demonstrated the simplification of a 20-story building using a generalized procedure tailored for large-scale structures [12]. A foundational analytical formulation was later developed for estimating high-rise buildings' natural frequencies and mode shapes during the early design stages, reducing each floor's DoF to three [13]. A matrix-based analytical strategy was also proposed for friction-damped systems. DoFs were categorized into primary and subsidiary sets, with frictional elements modeled using four DoFs, resulting in condensed primary matrices [14]. In seismic risk analysis, a practical method was presented that reduces both the DoF of the FE model and the number of required nonlinear time history analyses [15]. Lastly, a heuristic algorithm was introduced to efficiently derive the mass and stiffness matrices and calculate natural frequencies with minimal input data, offering considerable time savings over traditional FE approaches [16].

In this study, an optimal linear shear-type model of a 20-story benchmark building is developed based on the work of Ohotori et al. [17], which exhibits high accuracy. The process begins with constructing a preliminary linear shear-type model by estimating the story stiffness values of each story; however, this preliminary model exhibits significant discrepancies compared to the FE model. To address this, the stiffness coefficient is calibrated such that the fundamental frequency of the preliminary model aligns with that of the FE counterpart.

An optimization problem is formulated using a weighted sum approach to further enhance accuracy. Two objective functions are introduced to minimize frequency deviations between the preliminary and FE model: one uses equal weighting, and the other employs weighting based on modal participation percentages. The design variables are the story stiffness values, with the search domain defined as [2/3k, 4/3k], where k denotes the stiffness of each story in the preliminary model. Upon solving the optimization problem for both objective functions, optimized story stiffness values are obtained, which are then used to construct optimized shear-type models. These optimized models are then evaluated with the FE model based on frequencies and dynamic time-history response.

The floor masses are identical to those used by Ohotori et al., and Rayleigh damping is applied with a 2% damping ratio for the first and fifth modes. For validation through time-history analysis, both the optimized and FE models are subjected to the same input excitations from four benchmark ground motions: two far-field records (El Centro and Hachinohe) and two near-field records (Kobe and Northridge). The models' responses are compared in terms of Peak and Root-Mean-Square (RMS) displacement, velocity, and absolute acceleration values at the roof level.

2. STRUCTURE

2.1. Description of Case Study

This study utilizes the 20-story benchmark building, initially developed for the SAC Phase II Steel Project. Although never built, the structure was designed in compliance with seismic codes and serves as a representative high-rise building typical of those in Los Angeles, California. It was chosen as a benchmark case in SAC research to facilitate consistent and comparative evaluation of structural performance [17]. Detailed specifications of the structure are provided in [18]. The FE model of the benchmark building used in this study is based on the version of Farzam [19], which achieves significant accuracy compared to the model presented by Ohtori et al. Figure 1 illustrates the schematic layout, plan view, and connection details of the 20-story building, along with the specifications for the beams and columns.

2.2. Preliminary Shear-Type Model

A ROM is constructed as a linear shear-type model to reduce the number of DoF of the FE model. In this reduction, the original 20-story FE model, initially comprising 378 DoF, is reduced into a 20-DoF shear-type model. This is accomplished by assuming each floor behaves as a rigid diaphragm, with all horizontal displacements concentrated into a single DoF per floor. Masses are lumped at each floor level, resulting in a diagonal mass matrix. Story stiffness values are computed for each floor by aggregating the stiffness contributions of all beams and columns on that floor. Since the structural model assumes linear behavior, the superposition principle applies, allowing stiffnesses to be summed directly. With both mass and stiffness matrices defined, the damping matrix is also derived accordingly. Figure 2 presents a generalized N-story shear-type model under seismic excitation.

To estimate story stiffness in moment-resisting frames, a simplified method based on engineering judgment is proposed in [8] for developing linear shear-type models of benchmark buildings. In such frames, lateral stiffness can be approximated by relating lateral force to the corresponding lateral displacement.

The total lateral displacement combines column and beam deformations for structures designed with strong columns and weak beams, with approximate contributions of 40% and 60%, respectively. Accordingly, the story stiffness can be estimated using the following Equation:

$$K = \sum_{columns} \frac{4.8EI_c}{h^3} \tag{1}$$

In Eq. (1), E denotes the modulus of elasticity of the columns, Ic is the moment of inertia of the column cross-section, and h represents the story height. The 20-story benchmark structure is a moment-resisting frame system based on a strong-column/weak-beam design philosophy. Thus, Eq. (1) is employed to compute the lateral stiffness at each story. However, using a coefficient of 4.8 in Eq. (1) leads to a noticeable mismatch between the first frequency of the shear-type model and that of the FE model. Since accurately matching the first frequency is crucial for developing a reliable preliminary model, the coefficient was explicitly adjusted for the 20-story structure. An adjusted coefficient of 2.168 was adopted, resulting in an exact alignment between the first frequencies of the preliminary and the FE models.

A closer initial match between the preliminary and FE model, particularly in the first frequency, reduces the computational effort required by the Genetic Algorithm during optimization.

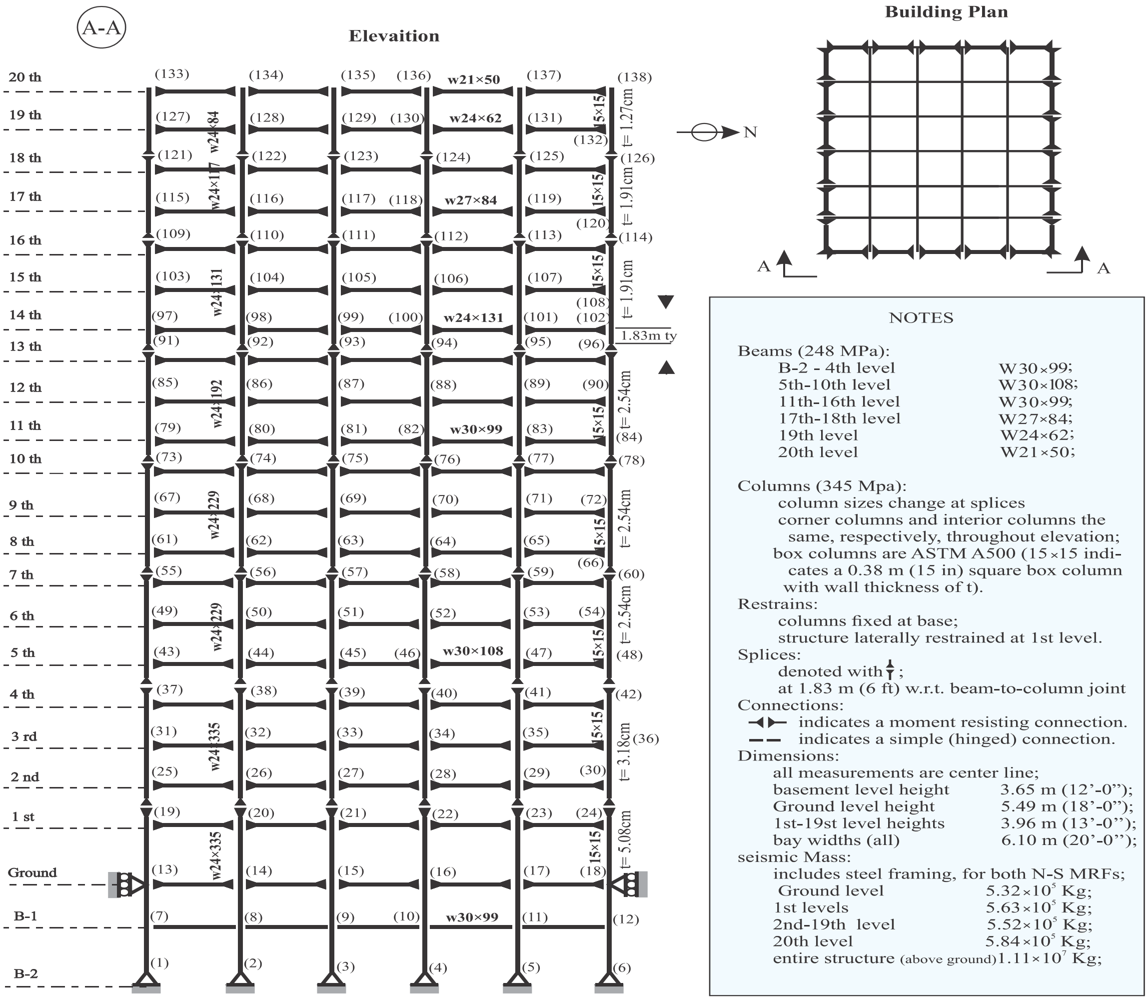

Figure 1: 20-story benchmark building north–south moment-resisting frame

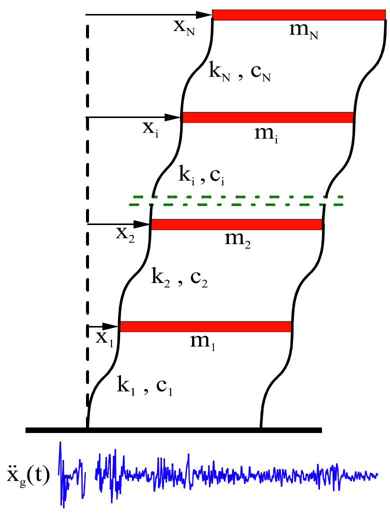

Figure 2: Sketch of an N-story shear-type model under earthquake excitation

3. GENETIC OPTIMIZATION ALGORITHM

Mathematical programming and metaheuristics are popular methods for optimizing problems. Mathematical programming offers fast convergence and high accuracy. However, their reliance on gradients and suitable starting points has led them to adopt metaheuristic algorithms. These algorithms explore and exploit solutions to find global or near-global optima, offering reasonable solutions promptly, even if they are not always absolute optima [20]. These algorithms are suitable for problems such as structural design, where the objective function and constraints can be complex, multivariable, and multi-objective [21]. Metaheuristic algorithms commonly used for structural optimization include Charged System Search (CSS), Ray Optimization (RO), Colliding Bodies Optimization (CBO), Genetic Algorithm (GA), and Magnetic Charged System Search (MCSS) [22].

Genetic Algorithms (GAs) are versatile search techniques widely used to solve complex optimization problems. Their adaptability makes them especially suitable for addressing various real-world engineering challenges. Since their introduction in the early 1980s, GAs have been the subject of extensive research and development across numerous disciplines. In structural engineering, GAs have demonstrated strong performance in optimizing various design and analysis tasks [23]. Rooted in the theory of evolution introduced by Charles Darwin [24], GAs were first formalized by Holland in 1975, with significant contributions and enhancements later provided by researchers like Goldberg [25]. It should be noted that these algorithms are presented solely as mathematical tools. Their detailed characteristics, mechanisms, and implementation aspects are not discussed here. For a more comprehensive study of these algorithms in civil engineering, the reader is referred to [26].

4. MATHEMATICAL FORMULATION

To evaluate a structure's response to seismic excitation, it is essential to formulate the governing equations that describe its dynamic behavior. These equations provide the foundation for calculating the time-history response, with displacement being the most critical output in dynamic analysis. For a system with *n* DoF, the equation of motion under earthquake excitation is given by [27]:

$$M\ddot{x} + C\dot{x} + Kx = -M\,\Gamma\ddot{x}g \tag{2}$$

Here, M, C, and K represent the structure's mass, damping, and stiffness matrices of size $n\times n$. The vector x contains the displacement values for each DoF, while $\ddot{x}$g denotes the ground acceleration. The vector Γ (of size $n\times 1$) defines how the ground motion influences each DoF. Solving this equation yields the structural time history responses.

5. DEFINITION OF OPTIMIZATION PROBLEM

An optimization problem is formulated to identify a linear shear-type model of the benchmark building that closely approximates the dynamic characteristics of the FE model.

A GA combined with a weighted sum method is employed to solve this optimization problem. In this approach, multiple objective functions are combined into a single scalar function using weighting coefficients, where each coefficient reflects the relative importance of its corresponding objective. Objectives considered more critical are assigned larger weights [28]. The objective function for this problem is defined as follows:

$$\mathbf{f}(\mathbf{X}) = \sum_{i=1}^{j} w_i f_i(x) \tag{3}$$

In this optimization problem, the design variables are the story stiffnesses. The optimal stiffness for each story is determined by minimizing the defined objective function across the search space. The problem is unconstrained, and the objective function is specifically formulated to minimize the frequency difference between the preliminary model and the FE model. Because achieving close agreement in the lower modes, especially the first mode, is essential, the corresponding objective terms are assigned larger weights. Two weighting strategies are employed to construct the objective function. In the first approach, all modes are assigned uniform weights, treating all frequency discrepancies equally. In the second approach, the weighting is based on the modal participation percent derived from the FE model, giving more weight to modes that contribute more significantly to the structural response. This ensures that the optimization prioritizes alignment in the most dynamically influential modes. Lower mode numbers are assigned greater weights in the corresponding objective function, as they certainly have a more significant impact on the structural response.

The model obtained using the first objective function is referred to as model 1, while the one resulting from the second objective function is referred to as model 2. The search space for each story's stiffness is defined within the range [2/3 k, 4/3 k], where k represents the initial stiffness of that story in the preliminary shear-type model. Two optimized shear-type models are developed by solving the optimization problem under both weighting strategies. These models are subsequently compared to the FE model in terms of frequencies and time-history responses to determine the most accurate model. The story masses in the optimized models are identical to those used by Ohtori et al. The damping matrix is constructed using Rayleigh damping, with a 2% damping ratio assigned to the first and fifth modes.

6. RESULTS ANALYSIS AND COMPARISON

6.1 Frequency Analysis

In model 1, where the objective function applies equal weighting, the discrepancies of all frequencies relative to their counterparts in the FE model are reduced uniformly. In contrast, for model 2, where modal participation percent weights are applied to the objective function, the primary frequencies exhibit fewer discrepancies compared to the frequencies of the FE model. Table 1 presents the first five frequencies, comparing them with their respective values in the FE model.

Table 1: Comparison of frequencies between optimized and FE models

	Frequency (Hz)			
Model	Error (%)	Optimum	FE	No.Mode
Model 1	1.976	0.258	0.253	1
	6.128	0.674	0.718	2
	8.818	1.127	1.236	3
	10.269	1.564	1.743	4
	12.090	2.014	2.291	5
Model 2	1.581	0.257	0.253	1
	0	0.718	0.718	2
	0.161	1.234	1.236	3
	0	1.743	1.743	4
	2.662	2.230	2.291	5

Model 2 is undoubtedly the optimal choice. Models 1 and 2 are compared in terms of frequency with the FE and the preliminary model, as shown in Figure 3 and Figure 4 for the first five frequencies. The optimized model 2 achieves a 1.581% discrepancy for the first frequency, zero for the second, 0.161% for the third, zero for the fourth, and 2.662% for the fifth, demonstrating greater accuracy than the optimized model 1. Model 2 surpasses the preliminary model and model 1 in accuracy.

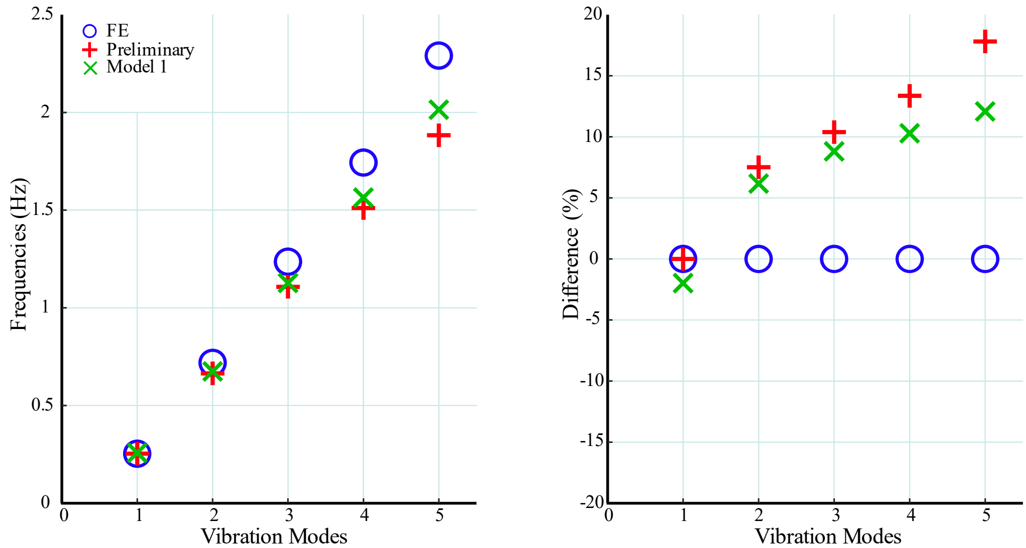

Figure 3: Comparison of frequencies between optimized model 1, the FE model, and the preliminary shear-type model

Figure 5 and Figure 6 illustrate the average absolute discrepancy across all modes during the optimization process and the convergence of preliminary model frequencies to the FE model frequencies for optimized models. In the upper figures, the vertical axis represents the percentage, while the horizontal axis denotes the number of analyses. The green curve indicates a trend in the average absolute discrepancy across all modes throughout the optimization process. The lower figures show the comparison of frequencies between the FE model and the optimized models for the first five frequencies throughout the optimization process. Across various analyses, the frequencies of the preliminary model rapidly converge to those of the FE model, indicating both a highly accurate preliminary model and an improvement in its accuracy as the optimization progresses, especially in model 2.

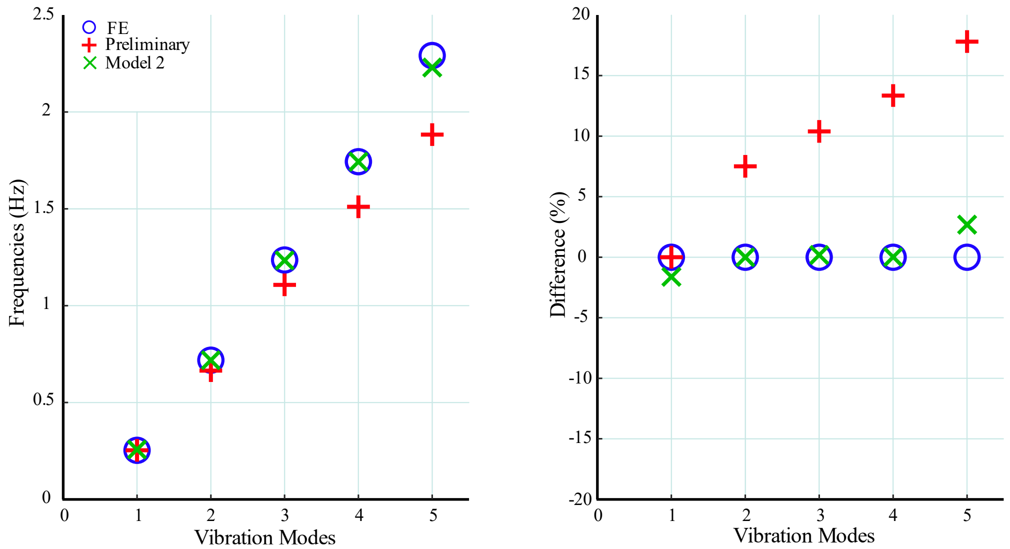

Figure 4: Comparison of frequencies between optimized model 2, the FE model, and the preliminary shear-type model

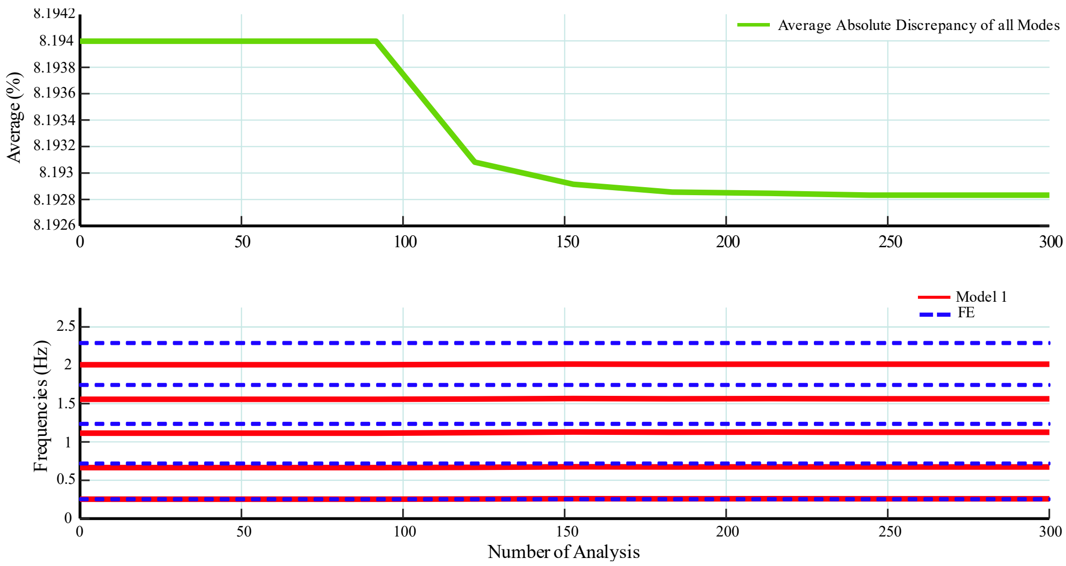

Figure 5: Iteration history of the optimization process in the optimized model 1

Although model 2, due to its weighted objective function that assigns higher weights to the lower modes and consequently achieves greater accuracy in these modes, reaches an average absolute discrepancy percentage of 0.0233 during the optimization process, model 1, with uniform weighting, reaches a discrepancy of 8.193.

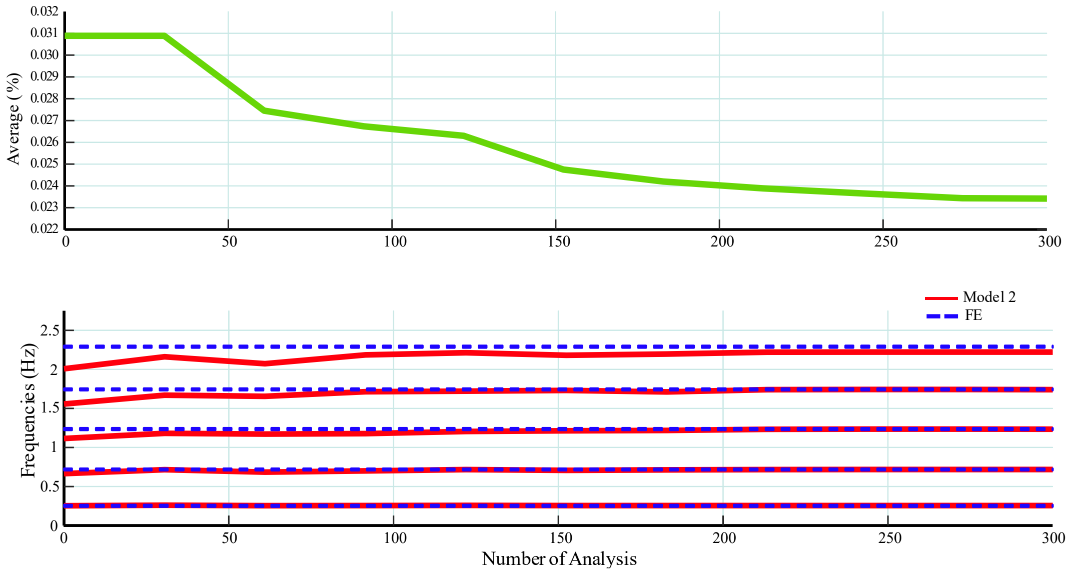

Figure 6: Iteration history of the optimization process in the optimized model 2

To further evaluate the effectiveness of the optimization process, the mode shapes of the optimized models 1 and 2 were compared with those of the FE model. The comparison revealed that the mode shapes of both optimized models closely align with those of the FE model, demonstrating the validity of the reduced-order representations. Moreover, the mode shapes of models 1 and 2 are almost identical, indicating that both weighting strategies yield comparable deformation patterns, despite differences in frequency accuracy. A more detailed examination shows that for the first three modes, the mode shapes obtained from model 2 are more accurate and closer to those of the FE model. In contrast, for the higher modes (i.e., modes 4 through 20), model 1 performs slightly better than model 2, with only minor differences observed. Notably, the first three modes collectively contribute 95.3% to the overall structural response, making their accurate representation particularly crucial. Figure 7 illustrates the first five mode shapes of all models, highlighting the strong agreement achieved through the optimization process.

6.2. Time-history analysis

This section analyzes the time-history responses of optimized models 1 and 2. The FE and optimized models are subjected to four benchmark earthquake excitations: two far-field records (El Centro and Hachinohe) and two near-field records (Kobe and Northridge). Their time-history response discrepancies are evaluated and compared. For this purpose, a time-history response has been obtained by subtracting the time-history response of the FE model from that of the optimized model. Subsequently, the resulting time-history response Peak and RMS were compared. The model with lower values is considered more optimal and accurate. This comparison approach is applied to roof responses, including displacement, velocity, and absolute acceleration, which are analyzed, with the results shown in Table 2. Since the Peak and RMS values of the resulting time-history responses for model 2 are lower, model 2 shows fewer discrepancies from the FE model and therefore demonstrates higher accuracy than model 1.

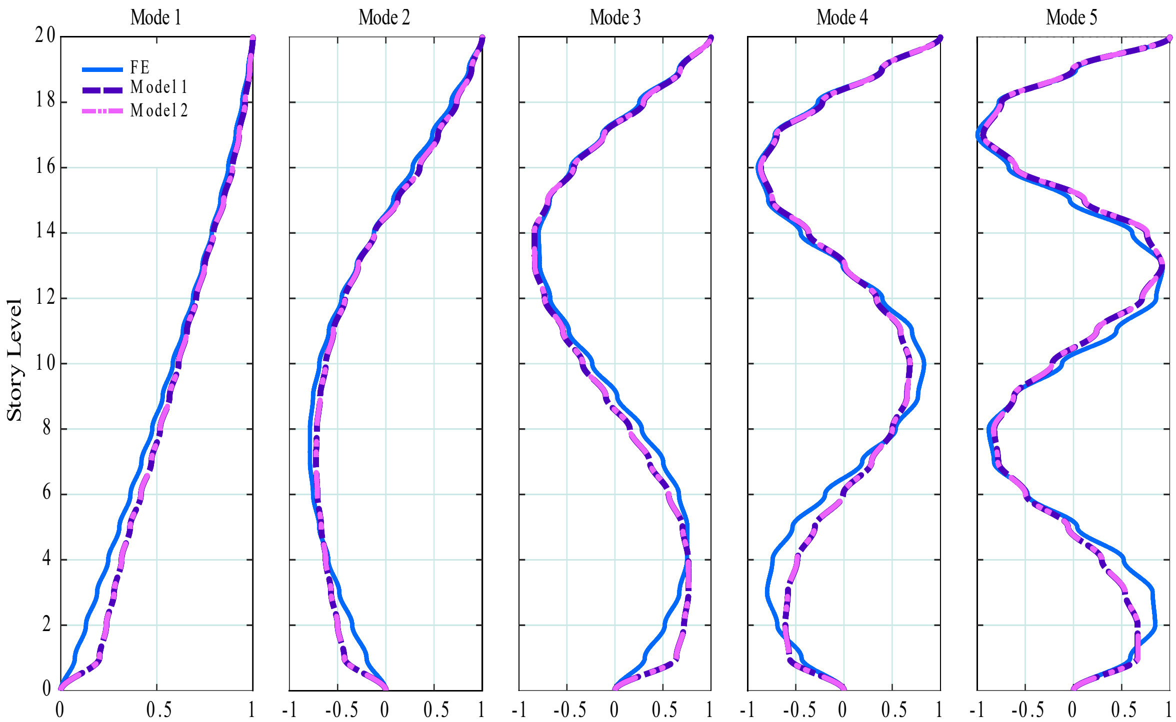

Figure 7: Comparison of the first five mode shapes of the optimized models with the FE model

The results reveal that the minor discrepancies in both Peak and RMS occur for displacement. In contrast, the most significant discrepancies are observed in absolute acceleration, particularly under the Kobe and Northridge earthquakes. The large discrepancy in Peak response is due to the rapid and intense oscillation in the absolute acceleration response curve, with slight misalignment in time between the FE and optimized models.

Table 2. Statistical comparison of response discrepancies between optimized models of the 9-story building to the FE model

Model		**Model 1**		**Model 2**	
Earthquake	**Response Type**	**Statistical**		**Statistical**	
		Peak	**RMS**	**Peak**	**RMS**
El Centro	Displacement(m)	0.155	0.054	0.083	0.037
	Velocity(m/s)	0.716	0.202	0.238	0.070
	Abs. Acceleration(m/s^2)	5.536	1.314	2.885	0.419
Hachinohe	Displacement(m)	0.078	0.033	0.046	0.024
	Velocity(m/s)	0.491	0.121	0.171	0.048
	Abs. Acceleration(m/s^2)	4.172	0.860	1.820	0.345
Kobe	Displacement(m)	0.447	0.176	0.224	0.114
	Velocity(m/s)	2.386	0.587	0.636	0.213
	Abs. Acceleration(m/s^2)	21.99	3.212	8.154	1.124
Northridge	Displacement(m)	0.569	0.203	0.245	0.129
	Velocity(m/s)	1.995	0.623	0.660	0.225
	Abs. Acceleration(m/s^2)	12.99	2.764	6.428	0.838

The Peak responses in the FE and optimized models do not coincide, resulting in a slight delay and contributing to more significant Peak discrepancies in absolute acceleration at the roof story. For example, under the Kobe earthquake, the Peak absolute acceleration in the FE model is 16.44 m/s², and the RMS is 2.538 m/s². In contrast, in model 1, the peak is 13.45 m/s², with a difference of 2.99 m/s², and the RMS is 2.246 m/s², with a difference of 0.292 m/s². The peak and RMS discrepancies in response to the proposed comparison approach are 21.99 m/s² and 3.212 m/s², respectively. If the comparison had been limited to evaluating the differences in Peak and RMS values without utilizing the proposed comparison approach, achieving accurate results from the time-history analysis regarding the precision of the optimized models would not have been possible.

Figure 8 presents the roof time-history responses of the optimized models and the FE model under the El Centro earthquake for up to 15 seconds. The time-history responses, including displacement, velocity, and absolute acceleration, are evaluated. Among these, both optimized models exhibit the highest agreement with the FE model in displacement and the least agreement in absolute acceleration. This discrepancy in acceleration is primarily attributed to the rapid and intense oscillation in the absolute acceleration response curve. Notably, model 2 demonstrates significantly better alignment and fewer discrepancies from the FE model and therefore demonstrates higher accuracy than model 1.

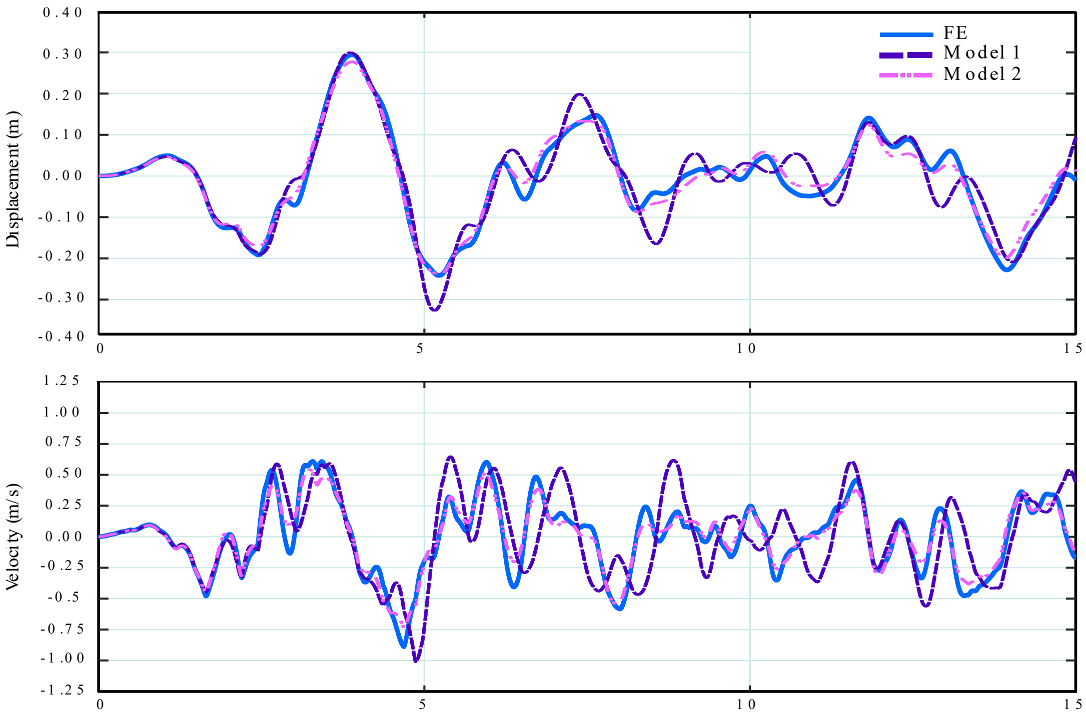

Figure 8: Comparison of roof time-history responses between the FE model and optimized models under the El Centro earthquake for up to 15 seconds

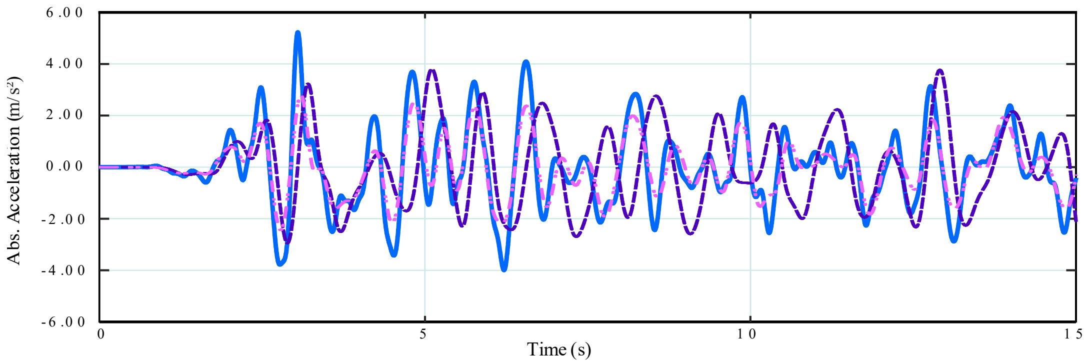

Figure 9: (continued)

7. CONCLUSION

This article addresses the reduction of degrees of freedom in the 20-story benchmark building to reduce computational time, cost, and effort. An optimized linear shear-type model was developed for this building using a genetic algorithm, exhibiting high accuracy compared to the FE model. Initially, a preliminary linear shear-type model was created by adjusting the coefficient in the stiffness equation, ensuring that the fundamental frequency matched that of the FE model. An optimization problem is defined to enhance the accuracy of the preliminary model, which is solved using a weighted sum approach. The design variables were the story stiffnesses of the preliminary model. Two objective functions were defined: one minimizes the difference between the frequencies of the FE model and the preliminary model with equal weighting, and the other uses modal participation percent weighting. By solving the optimization problem, the optimal stiffness for each story was determined according to the defined objective functions within the search space. The floor masses are identical to those used by Ohotori et al., and Rayleigh damping is applied with a 2% damping ratio for the first and fifth modes. The model obtained using the first objective function is referred to as model 1, while the one resulting from the second objective function is referred to as model 2.

The Frequency comparison between the FE and the optimized models showed that model 2 demonstrated significantly higher accuracy than model 1, indicating that the optimization process improved the accuracy of the preliminary model. By comparing the mode shapes of models 1 and 2 with those of the FE model, it was found that model 2 had slightly more minor discrepancies than model 1 in the first three modes relative to the FE model.

For time history analysis, the FE and optimized models were subjected to four benchmark earthquake excitations, and the displacement, velocity, and absolute acceleration responses at the roof story were compared. Peak and RMS differences in these responses, as evaluated by the proposed comparison approach, showed that model 2 had significantly fewer discrepancies than model 1. The model 2, presented in this article, closely represents the FE model. Model 2 can be used for dynamic analysis, control, and other structural and earthquake engineering purposes.

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